

Haversine formula

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The **haversine formula** is an equation important in [navigation](#), giving [great-circle distances](#) between two points on a [sphere](#) from their [longitudes](#) and [latitudes](#). It is a special case of a more general formula in [spherical trigonometry](#), the **law of haversines**, relating the sides and angles of spherical "triangles".

These names follow from the fact that they are customarily written in terms of the [haversine](#) function, given by $\text{hav}(\theta) = \sin^2(\theta/2)$. (The formulas could equally be written in terms of any multiple of the haversine, such as the older [versine](#) function (twice the haversine). Historically, the haversine had, perhaps, a slight advantage in that its maximum is one, so that [logarithmic](#) tables of its values could end at zero. These days, the haversine form is also convenient in that it has no coefficient in front of the \sin^2 function.)

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The haversine formula

For two points on a sphere (of radius R) with latitudes ϕ_1 and ϕ_2 , latitude separation $\Delta\phi = \phi_1 - \phi_2$, and longitude separation $\Delta\lambda$, where angles are in [radians](#), the distance d between the two points (along a [great circle](#) of the sphere; see [spherical distance](#)) is related to their locations by the formula:

(*the haversine formula*)

$$\text{havversin} \left(\frac{d}{R} \right) = \text{havversin}(\Delta\phi) + \cos(\phi_1) \cos(\phi_2) \text{havversin}(\Delta\lambda)$$

Let h denote $\text{havversin}(d/R)$, given from above. One can then solve for d either by simply applying the inverse haversine (if available) or by using the [arcsine](#) (inverse sine) function:

$$d = R \text{havversin}^{-1}(h) = 2R \arcsin \left(\sqrt{h} \right)$$

In the era before the [digital calculator](#), the use of detailed printed tables for the haversine/inverse-haversine and its [logarithm](#) (to aid multiplications) saved navigators from squaring sines, computing square roots, etc., a process both arduous and likely to exacerbate small errors (see also [versine](#)).

When using these formulae, care must be taken to ensure that h does not exceed 1 due to a [floating point](#) error (d is only [real](#) for h from 0 to 1). h only approaches 1 for *antipodal* points (on opposite sides of the sphere)—in this region, relatively large numerical errors tend to arise in the formula when finite precision is used. However, because d is then large (approaching πR , half the circumference) a small error is often not a major concern in this unusual case. (The formula above is sometimes written in terms of the [arctangent](#) function, but this suffers from similar numerical problems near $h = 1$.)

As described below, a similar formula can also be written in terms of cosines (sometimes called the *law of cosines*, not to be confused with the [law of cosines](#) for plane geometry) instead of haversines, but suffers numerical precision problems for the common case of small distances/angles, which render it unsuitable for serious use.

It should be noted that this formula is only an approximation when applied to the [Earth](#), because the Earth is not a perfect sphere: its radius R varies from 6356.78 [km](#) at the poles to 6378.14 km at the equator. There are small corrections, typically on the order of 0.1% (assuming the [geometric](#)

mean $R = 6367.45$ km is used everywhere), because of this slight ellipticity of the planet.

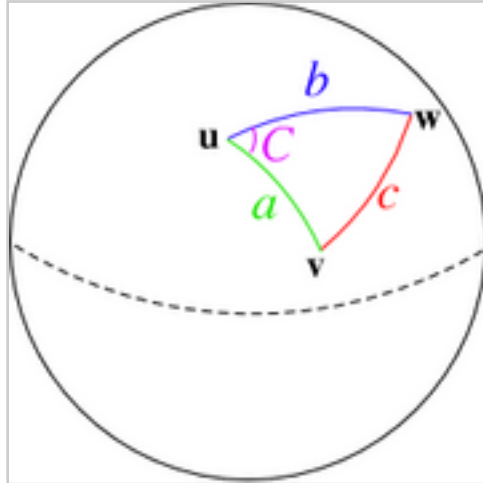
The law of haversines

Given a unit sphere, a "triangle" on the surface of the sphere is defined by the great circles connecting three points **u**, **v**, and **w** on the sphere. If the lengths of these three sides are a (from **u** to **v**), b (from **u** to **w**), and c (from **v** to **w**), and the angle of the corner opposite c is C , then the law of haversines states:

(the law of haversines)

$$\operatorname{hav}\sin(c) = \operatorname{hav}\sin(a - b) + \sin(a) \sin(b) \operatorname{hav}\sin(C)$$

Since this is a unit sphere, the lengths a , b , and c are simply equal to the angles (in radians) subtended by those sides from the center of the sphere (for a non-unit sphere, they are the distances divided by the radius).



Spherical triangle solved by the law of haversines.

In order to obtain the haversine formula of the previous section from this law, one simply considers the special case where **u** is the north pole, while **v** and **w** are the two points whose separation d is to be determined. In that case, a and b are $\pi/2 - \phi_{1,2}$ (i.e., $90^\circ - \text{latitude}$), C is the longitude separation $\Delta\lambda$, and c is the desired d/R . Noting that $\sin(\pi/2 - \phi) = \cos(\phi)$, the haversine formula immediately follows.

A proof of the law of haversines can be constructed as follows. Let \mathbf{u} , \mathbf{v} , and \mathbf{w} denote the unit vectors from the center of the sphere to those corners of the triangle. Then, the lengths (angles) of the sides are given by the dot products:

$$\begin{aligned}\cos(a) &= \mathbf{u} \cdot \mathbf{v} \\ \cos(b) &= \mathbf{u} \cdot \mathbf{w} \\ \cos(c) &= \mathbf{v} \cdot \mathbf{w}\end{aligned}$$

To get the angle C , we need the tangent vectors \mathbf{t}_a and \mathbf{t}_b at \mathbf{u} along the directions of sides a and b , respectively. For example, the tangent vector \mathbf{t}_a is the unit vector perpendicular to \mathbf{u} in the \mathbf{u} - \mathbf{v} plane, whose direction is given by the component of \mathbf{v} perpendicular to \mathbf{u} . This means:

$$\mathbf{t}_a = \frac{\mathbf{v} - \mathbf{u}(\mathbf{u} \cdot \mathbf{v})}{|\mathbf{v} - \mathbf{u}(\mathbf{u} \cdot \mathbf{v})|} = \frac{\mathbf{v} - \mathbf{u} \cos(a)}{\sin(a)}$$

where for the denominator we have used the Pythagorean identity $\sin^2(a) = 1 - \cos^2(a)$. Similarly,

$$\mathbf{t}_b = \frac{\mathbf{w} - \mathbf{u} \cos(b)}{\sin(b)}$$

Then, the angle C is given by:

$$\cos(C) = \mathbf{t}_a \cdot \mathbf{t}_b = \frac{\cos(c) - \cos(a) \cos(b)}{\sin(a) \sin(b)}$$

which leads to the (spherical) *law of cosines*, also called the *cosine rule for sides*:

(*spherical law of cosines*)

$$\cos(c) = \cos(a) \cos(b) + \sin(a) \sin(b) \cos(C)$$

As mentioned above, this formula is an ill-conditioned way of solving for c when c is small.

Instead, we substitute the identity that $\cos(\theta) = 1 - 2 \operatorname{hav}\sin(\theta)$, and also employ the addition identity $\cos(a - b) = \cos(a) \cos(b) + \sin(a) \sin(b)$, to obtain the law of haversines, above.

References

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- R. W. Sinnott, "Virtues of the Haversine", *Sky and Telescope* **68** (2), 159 (1984).
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- Romuald Ireneus 'Scibor-Marchocki, Spherical trigonometry, *Elementary-Geometry Trigonometry* web page (1997).
- W. Gellert, S. Gottwald, M. Hellwich, H. Kästner, and H. Küstner, *The VNR Concise Encyclopedia of Mathematics*, 2nd ed., ch. 12 (Van Nostrand Reinhold: New York, 1989).

External links

- JavaScript implementation of Haversine formula to find distance between two latitude/longitude points

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Category: [Spherical trigonometry](#)

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