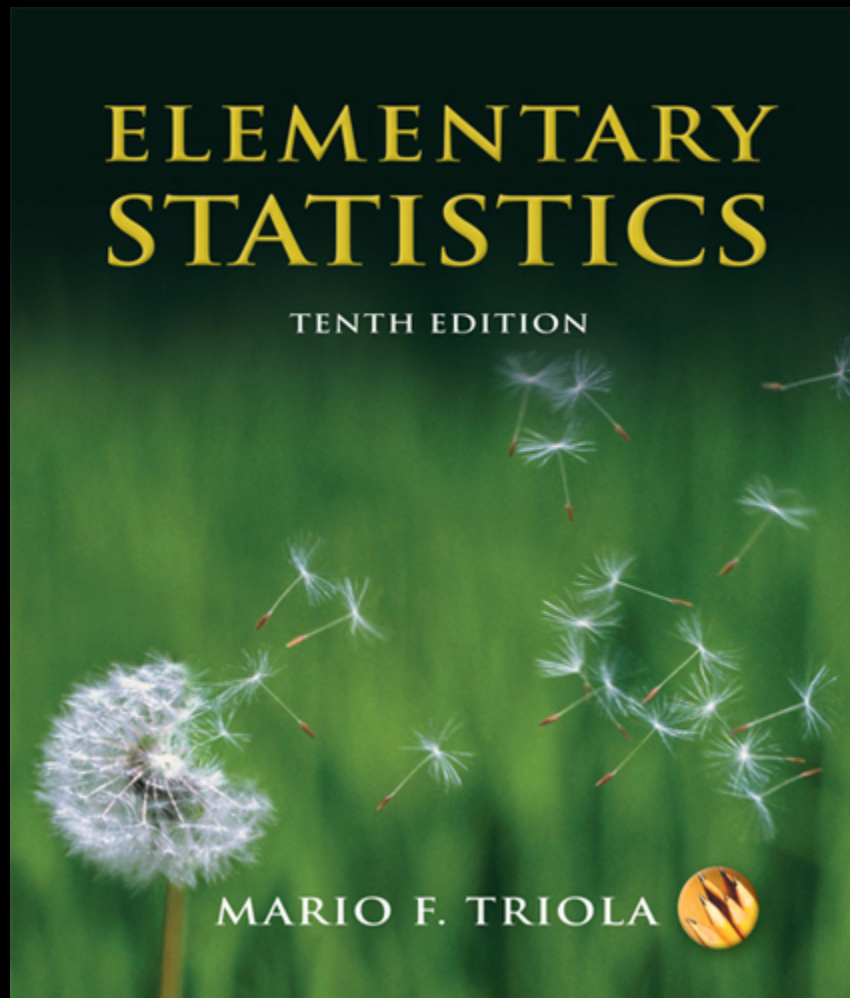


Lecture Slides



Elementary Statistics Tenth Edition

and the Triola Statistics Series

by Mario F. Triola

Chapter 4

Probability

4-1 Overview

4-2 Fundamentals

4-3 Addition Rule

4-4 Multiplication Rule: Basics

4-5 Multiplication Rule: Complements and Conditional Probability

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Section 4-1 Overview

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Overview

Rare Event Rule for Inferential Statistics:

If, under a given assumption, the probability of a particular observed event is extremely small, we conclude that the assumption is probably not correct.

Statisticians use the **rare event rule for inferential statistics**.



Section 4-2 Fundamentals

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Key Concept

This section introduces the basic concept of the **probability** of an event. Three different methods for finding probability values will be presented.

The most important objective of this section is to learn how to **interpret** probability values.

Definitions

❖ Event

any collection of results or outcomes of a procedure

❖ Simple Event

an outcome or an event that cannot be further broken down into simpler components

❖ Sample Space

for a procedure consists of all possible **simple** events; that is, the sample space consists of all outcomes that cannot be broken down any further

Notation for Probabilities

P - denotes a probability.

A , B , and C - denote specific events.

$P(A)$ - denotes the probability of event A occurring.

Basic Rules for Computing Probability

Rule 1: Relative Frequency Approximation of Probability

Conduct (or observe) a procedure, and count the number of times event A actually occurs. Based on these actual results, $P(A)$ is **estimated** as follows:

$$P(A) = \frac{\text{number of times } A \text{ occurred}}{\text{number of times trial was repeated}}$$

Basic Rules for Computing Probability - cont

Rule 2: Classical Approach to Probability (Requires Equally Likely Outcomes)

Assume that a given procedure has n different simple events and that **each of those simple events has an equal chance of occurring**. If event A can occur in s of these n ways, then

$$P(A) = \frac{s}{n} = \frac{\text{number of ways } A \text{ can occur}}{\text{number of different simple events}}$$

Basic Rules for Computing Probability - cont

Rule 3: Subjective Probabilities

$P(A)$, the probability of event A , is **estimated** by using knowledge of the relevant circumstances.

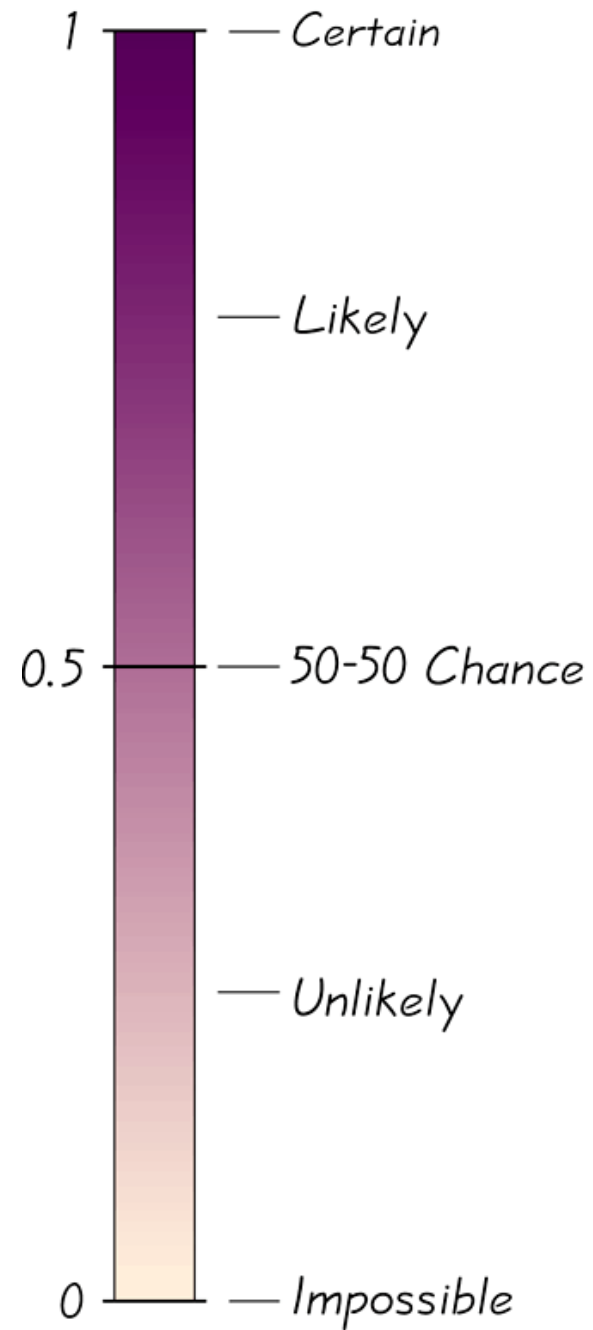
Law of Large Numbers

As a procedure is repeated again and again, the relative frequency probability (from Rule 1) of an event tends to approach the actual probability.

Probability Limits

- ❖ The probability of an impossible event is 0.
- ❖ The probability of an event that is certain to occur is 1.
- ❖ For any event A , the probability of A is between 0 and 1 inclusive.
That is, $0 \leq P(A) \leq 1$.

Possible Values for Probabilities



Definition

The complement of event A , denoted by \bar{A} , consists of all outcomes in which the event A does **not** occur.

Rounding Off Probabilities

When expressing the value of a probability, either give the **exact** fraction or decimal or round off final decimal results to three significant digits. (**Suggestion**: When the probability is not a simple fraction such as $2/3$ or $5/9$, express it as a decimal so that the number can be better understood.)

Definitions

The **actual odds against** event A occurring are the ratio $P(A)/P(\bar{A})$, usually expressed in the form of **a:b** (or “**a** to **b**”), where a and b are integers having no common factors.

The **actual odds in favor** of event A occurring are the reciprocal of the actual odds against the event. If the odds against A are **a:b**, then the odds in favor of A are **b:a**.

The **payoff odds** against event A represent the ratio of the net profit (if you win) to the amount bet.

$$\text{payoff odds against event } A = (\text{net profit}) : (\text{amount bet})$$

Recap

In this section we have discussed:

- ❖ **Rare event rule for inferential statistics.**
- ❖ **Probability rules.**
- ❖ **Law of large numbers.**
- ❖ **Complementary events.**
- ❖ **Rounding off probabilities.**
- ❖ **Odds.**



Section 4-3 Addition Rule

Created by Tom Wegleitner, Centreville, Virginia



Key Concept

The main objective of this section is to present the **addition rule** as a device for finding probabilities that can be expressed as $P(A \text{ or } B)$, the probability that either event A occurs or event B occurs (or they both occur) as the single outcome of the procedure.

Definition

Compound Event

any event combining 2 or more simple events

Notation

$P(A \text{ or } B) = P$ (in a single trial, event A occurs or event B occurs or they both occur)

General Rule for a Compound Event

When finding the probability that event A occurs or event B occurs, find the total number of ways A can occur and the number of ways B can occur, but **find the total in such a way that no outcome is counted more than once.**

Compound Event

Formal Addition Rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

where $P(A \text{ and } B)$ denotes the probability that A and B both occur at the same time as an outcome in a trial or procedure.

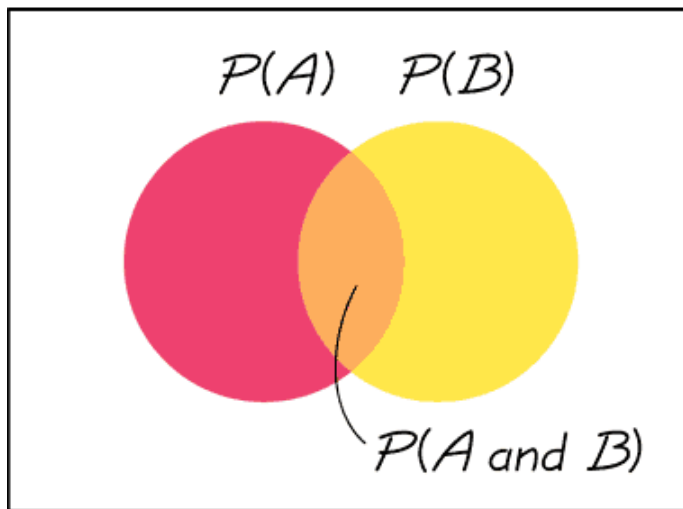
Intuitive Addition Rule

To find $P(A \text{ or } B)$, find the sum of the number of ways event A can occur and the number of ways event B can occur, **adding in such a way that every outcome is counted only once**. $P(A \text{ or } B)$ is equal to that sum, divided by the total number of outcomes in the sample space.

Definition

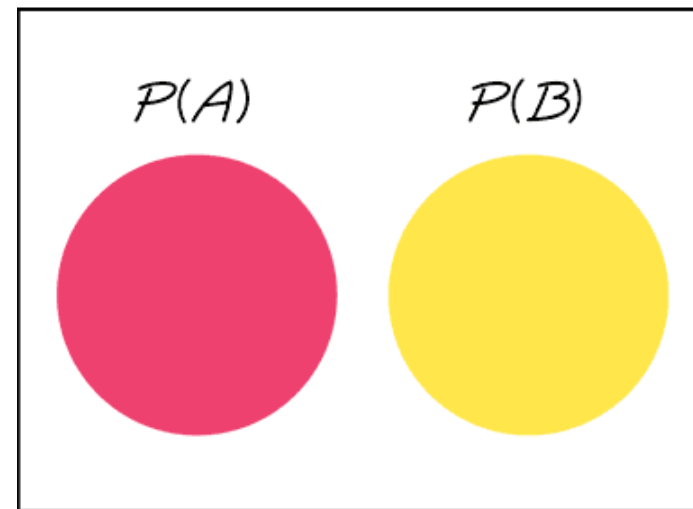
Events A and B are **disjoint** (or **mutually exclusive**) if they cannot occur at the same time. (That is, disjoint events do not overlap.)

Total Area = 1



Venn Diagram for Events That Are Not Disjoint

Total Area = 1



Venn Diagram for Disjoint Events

Complementary Events

$P(A)$ and $P(\bar{A})$
are disjoint

It is impossible for an event and its complement to occur at the same time.

Rules of Complementary Events

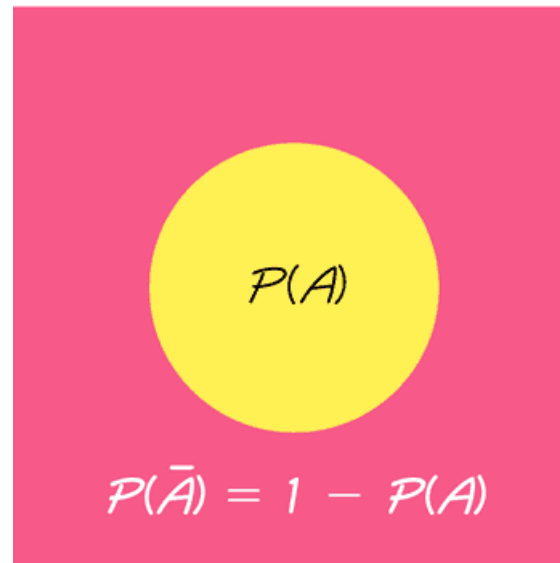
$$P(A) + P(\bar{A}) = 1$$

$$P(\bar{A}) = 1 - P(A)$$

$$P(A) = 1 - P(\bar{A})$$

Venn Diagram for the Complement of Event A


Total Area = 1



Recap

In this section we have discussed:

- ❖ **Compound events.**
- ❖ **Formal addition rule.**
- ❖ **Intuitive addition rule.**
- ❖ **Disjoint events.**
- ❖ **Complementary events.**



Section 4-4

Multiplication Rule:

Basics

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Key Concept

If the outcome of the first event A somehow affects the probability of the second event B , it is important to adjust the probability of B to reflect the occurrence of event A .

The rule for finding $P(A \text{ and } B)$ is called the multiplication rule.

Notation

$P(A \text{ and } B) =$

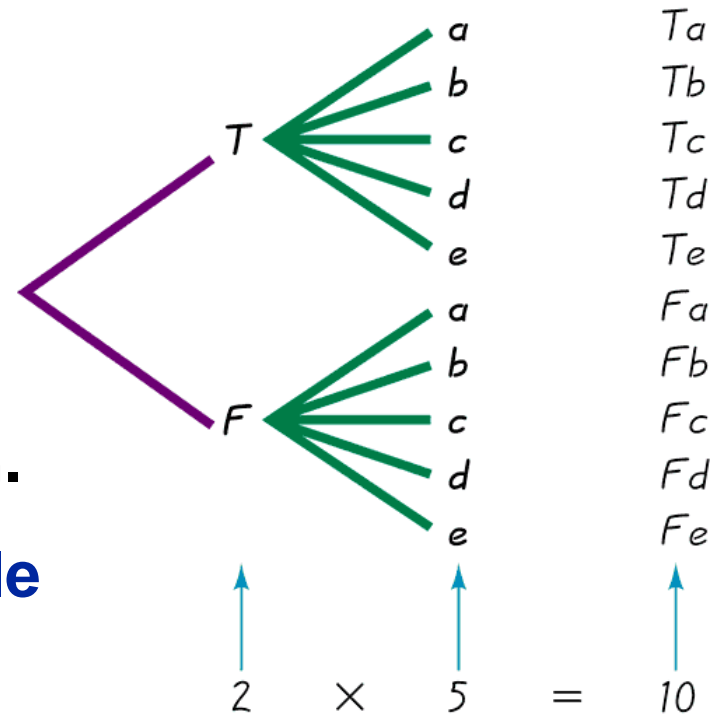
**$P(\text{event } A \text{ occurs in a first trial and}$
 $\text{event } B \text{ occurs in a second trial})$**

Tree Diagrams

A **tree diagram** is a picture of the possible outcomes of a procedure, shown as line segments emanating from one starting point. These diagrams are helpful if the number of possibilities is not too large.

This figure summarizes the possible outcomes for a true/false followed by a multiple choice question.

Note that there are 10 possible combinations.



Key Point – Conditional Probability

The probability for the second event B should take into account the fact that the first event A has already occurred.

Notation for Conditional Probability

$P(B|A)$ represents the probability of event B occurring after it is assumed that event A has already occurred (read $B|A$ as “ B given A .”)

Definitions

Independent Events

Two events A and B are **independent** if the occurrence of one does not affect the probability of the occurrence of the other. (Several events are similarly independent if the occurrence of any does not affect the probabilities of occurrence of the others.) If A and B are not independent, they are said to be **dependent**.

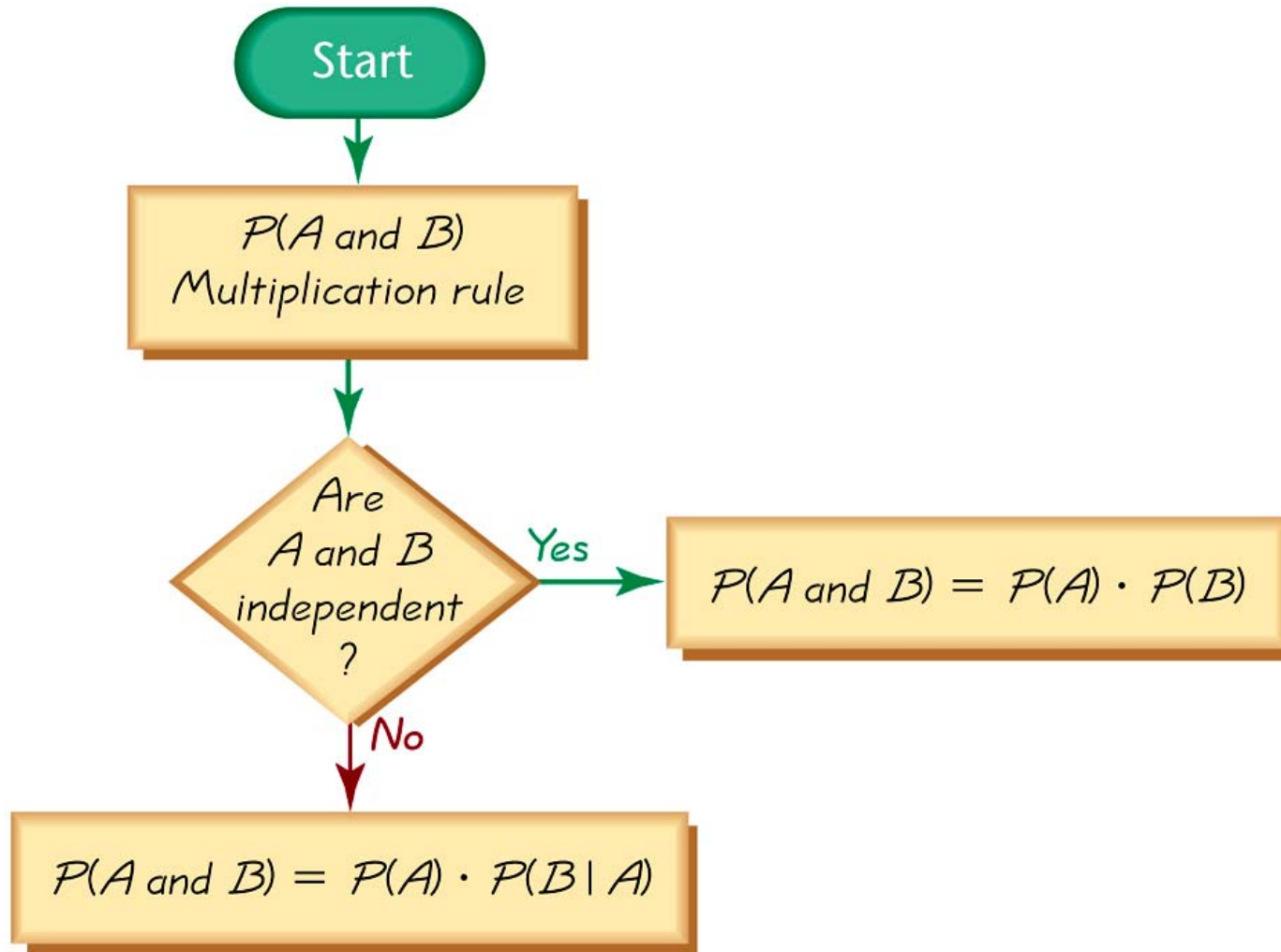
Formal Multiplication Rule

- ❖ $P(A \text{ and } B) = P(A) \cdot P(B|A)$
- ❖ Note that if A and B are independent events, $P(B|A)$ is really the same as $P(B)$.

Intuitive Multiplication Rule

When finding the probability that event A occurs in one trial and event B occurs in the next trial, multiply the probability of event A by the probability of event B , but be sure that the probability of event B takes into account the previous occurrence of event A .

Applying the Multiplication Rule



Small Samples from Large Populations

If a sample size is no more than 5% of the size of the population, treat the selections as being **independent** (even if the selections are made without replacement, so they are technically dependent).

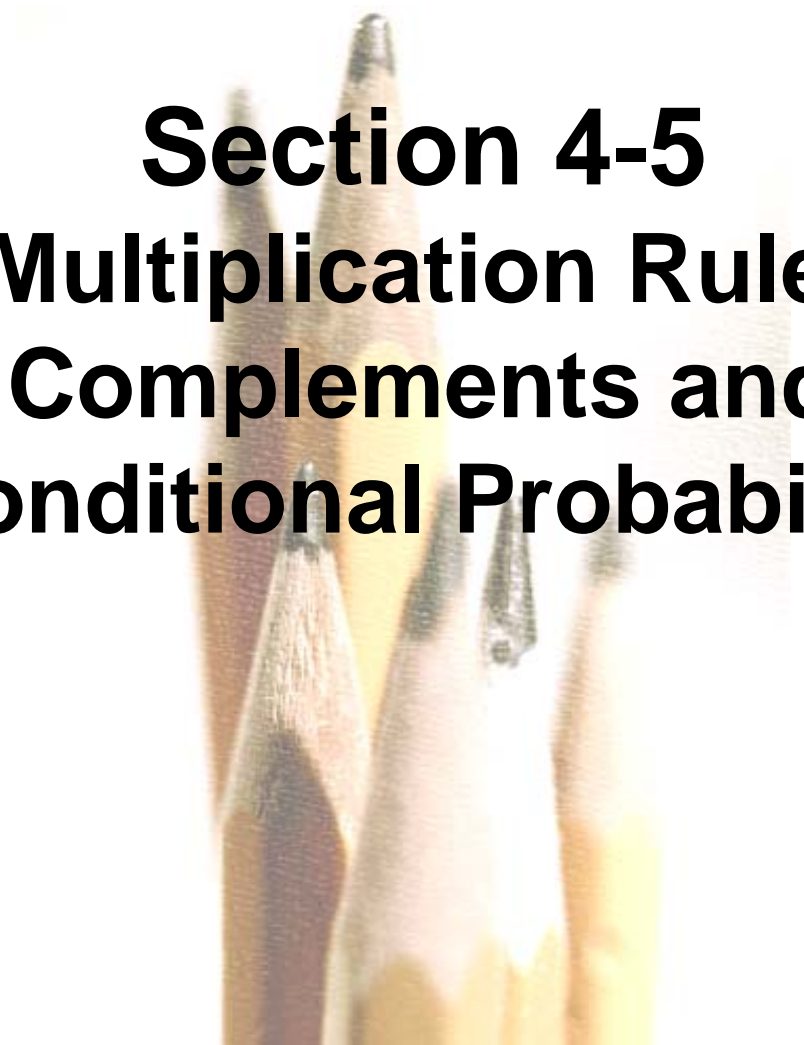
Summary of Fundamentals

- ❖ In the addition rule, the word “or” in $P(A \text{ or } B)$ suggests addition. Add $P(A)$ and $P(B)$, being careful to add in such a way that every outcome is counted only once.
- ❖ In the multiplication rule, the word “and” in $P(A \text{ and } B)$ suggests multiplication. Multiply $P(A)$ and $P(B)$, but be sure that the probability of event B takes into account the previous occurrence of event A .

Recap

In this section we have discussed:

- ❖ Notation for $P(A \text{ and } B)$.
- ❖ Tree diagrams.
- ❖ Notation for conditional probability.
- ❖ Independent events.
- ❖ Formal and intuitive multiplication rules.



Section 4-5

Multiplication Rule: Complements and Conditional Probability

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Key Concept

In this section we look at the probability of getting **at least one** of some specified event; and the concept of conditional probability which is the probability of an event given the additional information that some other event has already occurred.

Complements: The Probability of “At Least One”

- ❖ “At least one” is equivalent to “one or more.”
- ❖ The complement of getting at least one item of a particular type is that you get **no** items of that type.

Key Principle

To find the probability of **at least one** of something, calculate the probability of **none**, then subtract that result from 1. That is,

$$P(\text{at least one}) = 1 - P(\text{none}).$$

Definition

A **conditional probability** of an event is a probability obtained with the additional information that some other event has already occurred. $P(B|A)$ denotes the conditional probability of event B occurring, given that event A has already occurred, and it can be found by dividing the probability of events A and B both occurring by the probability of event A :

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Intuitive Approach to Conditional Probability

The conditional probability of B given A can be found by assuming that event A has occurred and, working under that assumption, calculating the probability that event B will occur.

Recap

In this section we have discussed:

- ❖ **Concept of “at least one.”**
- ❖ **Conditional probability.**
- ❖ **Intuitive approach to conditional probability.**



Section 4-6

Probabilities Through

Simulations

Created by Tom Wegleitner, Centreville, Virginia



Key Concept

In this section we introduce a very different approach for finding probabilities that can overcome much of the difficulty encountered with the formal methods discussed in the preceding sections of this chapter.

Definition

A **simulation** of a procedure is a process that behaves the same way as the procedure, so that similar results are produced.

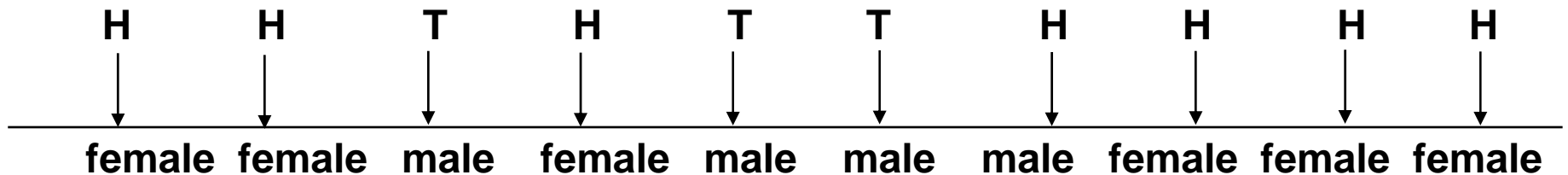
Simulation Example

Gender Selection When testing techniques of gender selection, medical researchers need to know probability values of different outcomes, such as the probability of getting at least 60 girls among 100 children. Assuming that male and female births are equally likely, describe a simulation that results in genders of 100 newborn babies.

Simulation Examples

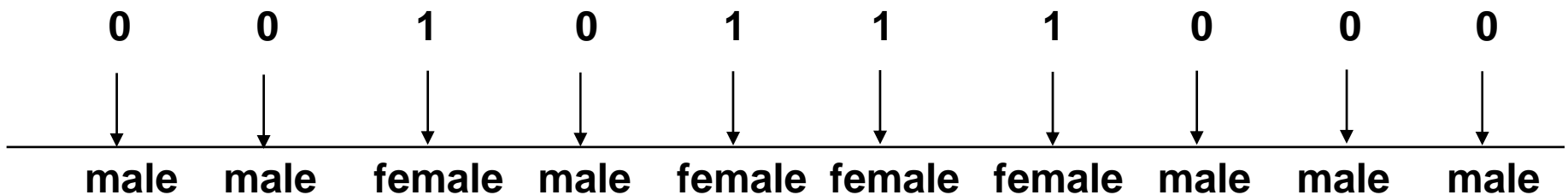
Solution 1:

- ❖ Flipping a fair coin 100 times where
heads = female
tails = male



Solution 2:

- ❖ Generating 0's and 1's with a computer or calculator where
0 = male
1 = female



Random Numbers

In many experiments, **random numbers** are used in the simulation of naturally occurring events. Below are some ways to generate random numbers.

- ❖ A table of random of digits
- ❖ STATDISK
- ❖ Minitab
- ❖ Excel
- ❖ TI-83 Plus calculator

Random Numbers - cont

STATDISK

Row	1 Ran...
1	7
2	8
3	16
4	38
5	42
6	46
7	68
8	68
9	104
10	117
11	140
12	195
13	204
14	244
15	271
16	274

Minitab

↓	C1	C2
1	38	
2	48	
3	59	
4	71	
5	101	
6	107	
7	122	
8	129	
9	153	
10	153	
11	163	

Random Numbers - cont

Excel

	A
1	15
2	3
3	15
4	362
5	164
6	184
7	158
8	59
9	143
10	85
11	134

TI-83 Plus calculator

```
randInt(1,365,25  
→L1  
{79 206 340 133...  
SortA(L1)  
Done  
L1  
{17 34 46 70 79...
```


Recap

In this section we have discussed:

- ❖ **The definition of a simulation.**
- ❖ **How to create a simulation.**
- ❖ **Ways to generate random numbers.**



Section 4-7 Counting

Created by Tom Wegleitner, Centreville, Virginia



Key Concept

In many probability problems, the big obstacle is finding the total number of outcomes, and this section presents several methods for finding such numbers without directly listing and counting the possibilities.

Fundamental Counting Rule

For a sequence of two events in which the first event can occur m ways and the second event can occur n ways, the events together can occur a total of $m \cdot n$ ways.

Notation

The **factorial symbol !** denotes the product of decreasing positive whole numbers.

For example,

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24.$$

By special definition, $0! = 1$.

Factorial Rule

A collection of n different items can be arranged in order $n!$ different ways.
(This **factorial rule** reflects the fact that the first item may be selected in n different ways, the second item may be selected in $n - 1$ ways, and so on.)

Permutations Rule (when items are all different)

Requirements:

1. There are n different items available. (This rule does not apply if some of the items are identical to others.)
2. We select r of the n items (without replacement).
3. We consider rearrangements of the same items to be different sequences. (The permutation of **ABC** is different from **CBA** and is counted separately.)

If the preceding requirements are satisfied, the number of **permutations** (or sequences) of r items selected from n available items (without replacement) is

$${}_n P_r = \frac{n!}{(n - r)!}$$

Permutations Rule

(when some items are **identical to others**)

Requirements:

1. There are n items available, and some items are identical to others.
2. We select all of the n items (without replacement).
3. We consider rearrangements of distinct items to be different sequences.

If the preceding requirements are satisfied, and if there are n_1 alike, n_2 alike, . . . n_k alike, the number of **permutations** (or sequences) of all items selected without replacement is

$$\frac{n!}{n_1! \cdot n_2! \cdot \dots \cdot n_k!}$$

Combinations Rule

Requirements:

1. There are n different items available.
2. We select r of the n items (without replacement).
3. We consider rearrangements of the same items to be the same. (The combination of **ABC** is the same as **CBA**.)

If the preceding requirements are satisfied, the number of **combinations** of r items selected from n different items is

$${}_n C_r = \frac{n!}{(n - r)! r!}$$

Permutations versus Combinations

When different orderings of the same items are to be counted separately, we have a permutation problem, but when different orderings are not to be counted separately, we have a combination problem.

Recap

In this section we have discussed:

- ❖ The fundamental counting rule.
- ❖ The factorial rule.
- ❖ The permutations rule (when items are all different).
- ❖ The permutations rule (when some items are identical to others).
- ❖ The combinations rule.