**Problem: Find** $\displaystyle\int \frac{\sec(x)}{\cot(x)}\,dx$

**Mathcad solution:** $\displaystyle\int \frac{\sec(x)}{\cot(x)}\,dx \rightarrow \frac{1}{\cos(x)}$

**Integrating by parts**

$$\int \frac{\sec(x)}{\cot(x)}\,dx = \int \sin(x)\cdot\sec(x)^2\,dx$$

$u = \sin(x)$ $\quad du = \cos(x)\cdot dx$

$dv = \sec(x)^2\cdot dx$ $\quad v = \dfrac{\sin(x)}{\cos(x)}$ ***SEE BELOW**

$$\int u\,dv = uv - \int v\,du$$

$$\int \sin(x)\cdot\sec(x)^2\,dx = \sin(x)\cdot\frac{\sin(x)}{\cos(x)} - \int \frac{\sin(x)}{\cos(x)}\cdot\cos(x)\,dx = \sin(x)\cdot\frac{\sin(x)}{\cos(x)} - \int \sin(x)\,dx = \sin(x)\cdot\frac{\sin(x)}{\cos(x)} + \cos(x) = \frac{1-\cos(x)^2}{\cos(x)} + \frac{\cos(x)^2}{\cos(x)} = \frac{1}{\cos(x)}$$

$$\int \frac{\sec(x)}{\cot(x)}\,dx = \frac{1}{\cos(x)}$$

***show:** $\displaystyle\int \sec(x)^2\,dx = \frac{\sin(x)}{\cos(x)}$ **by showing:** $\displaystyle\frac{d}{dx}\frac{\sin(x)}{\cos(x)} = \sec(x)^2$ remembering: $\displaystyle\frac{d}{dx}\frac{u}{v} = \frac{1}{v}\cdot\left(\frac{d}{dx}u\right) - \frac{u}{v^2}\cdot\left(\frac{d}{dx}v\right)$

$$\frac{d}{dx}\frac{\sin(x)}{\cos(x)} = \frac{1}{\cos(x)}\cdot\left(\frac{d}{dx}\sin(x)\right) - \frac{\sin(x)}{\cos(x)^2}\cdot\left(\frac{d}{dx}\cos(x)\right) = \frac{1}{\cos(x)}\cdot(\cos(x)) + \frac{\sin(x)}{\cos(x)^2}\cdot(\sin(x)) = \frac{\cos(x)^2}{\cos(x)^2} + \frac{\sin(x)^2}{\cos(x)^2} = \frac{\cos(x)^2+\sin(x)^2}{\cos(x)^2} = \frac{1}{\cos(x)^2} = \sec(x)^2$$